The optimum quantity of debt

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MOTIVATION - research question

What is the welfare maximing level of public debt?

- Earlier studies used deterministic representative agent models
- They arrived at two main results:
  1. public debt is welfare improving only if taxes are costly
  2. optimal level either indeterminate or set by initial conditions
- In AM's model, public debt introduces tradeoffs:
  - \+ enhances household (hh) consumption smoothing
  - — requires costly taxation
  - — crowds out productive capital and increases interest rate

MODEL - environment

- AM present two models based on Aiyagari [1994, 1995]:
  - no aggregate but individual risk (stochastic labor productivity)
  - perfectly competitive firms employ capital and labor
  - market incompleteness: risk free asset, borrowing constraints
  - precautionary savings as equilibrium outcome

1. Reduced model: Aiyagari with
   - growth
   - government debt
   - exogenous and wasteful government consumption
   - lump sum taxes
   - exogenous labor supply

   $\rightarrow$ taxation has no insurance and incentive effects
2. Benchmark model: Reduced model with
   - proportional income tax
   - endogenous labor supply

   $\rightarrow$ will be adapted to US to study welfare effects of public debt

REDUCED MODEL - lump sum tax, exogenous labor supply

- Technology:
  - stochastic labor producvitity e_t; normalized $E(e_t) = 1$
  - labor augmenting technological progress: $z_t = z(1+g)^t$
  - growth adjustment: $Y_t = F(K_t, z_t N_t)$
  - capital depreciates at rate δ
- Households:

$$\max_{c_t, a_{t+1}} E\left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\nu}}{1-\nu}\right]$$

s.t.

$$c_t + a_{t+1} \leq (1+r)a_t + w_t e_t - T_t$$

$$c_t \geq 0; a_t \geq 0; a_0, e_0 \text{ given}$$

- Government budget: $G_t + rB_t = B_{t+1} - B_t + T_t$
- Asset market: $A_t = K_t + B_t$ (A_t: per capita assets)

REDUCED MODEL - in stationary steady state

- Along balanced growth path:
  - r constant
  - Y, K, T, B, A (in per capita terms) and w grow at g
  - lower case/wiggled letters denote variables divided by Y
- Households:

$$\max_{\tilde{c}_t, \tilde{a}_{t+1}} E\left[(Y_0)^{1-\nu} \sum_{t=0}^{\infty} \left[\beta(1+g)^{1-\nu}\right]^t \frac{\tilde{c}_t^{1-\nu}}{1-\nu}\right]$$

s.t.

$$\tilde{c}_t + (1+g)\tilde{a}_{t+1} \leq (1+r)\tilde{a}_t + \tilde{w}_t e_t - \tau$$

$$\tilde{c}_t \geq 0; \tilde{a}_t \geq 0; \ \tilde{a}_0, e_0, Y_0 \text{ given}$$

- Government budget: $\gamma + (r-g)b = \tau \qquad (\gamma = G_t/Y_t)$
- Asset market: $\bar{a} = k + b \qquad (\bar{a} = A_t/Y_t)$

REDUCED MODEL - CE in stationary steady state

- In this environment, a competitive equilibrium is a set of
  - hh policy functions $\mathscr{A}'(a, e)$ and $\mathscr{C}(a, e)$
  - factor inputs L and K
  - factor prices w and r
  - government debt B
  - taxes T

  such that
  - the equilibrium distribution of hhs over the state space $\lambda(a, e)$ associated with $\mathscr{A}'(a, e)$ and $\pi(e'|e)$ is stationary
  - given r, w, T: $\mathscr{A}'$,$\mathscr{C}$ maximize the hh problem (s.t. constraints)
  - given r, w: firms choose K and L so they get paid their MPs
  - hh savings supply equals demand by firms and government
  - hh labor supply equals demand by firms
  - government budget is satisfied
  - goods market clears

REDUCED MODEL - interest rate determination

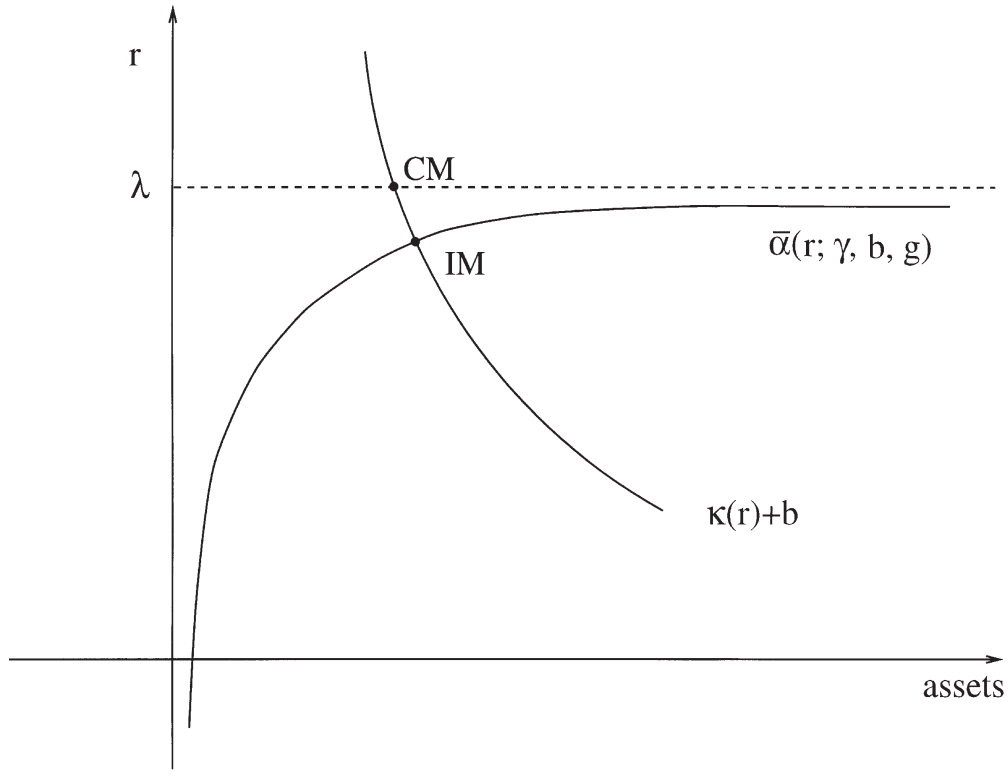

Fig. 1. Interest rate determination.

- Note: $\lambda \equiv \frac{(1+g)^{\nu}}{\beta} - 1$ (CM asset demand)
- Asset demand:
  - $\mathscr{A}'$ gives stationary distribution of assets, π of shocks
  - integrate to get $\overline{\alpha}(r; \gamma, b, g)$
- Asset supply: $\kappa(r) + b$ (k is function of r via MPK)

REDUCED MODEL - the effect of increasing public debt

- Relative to CM: interest rate lower and capital stock higher

  $\Rightarrow$ government debt is not neutral and has two effects
- To see them:
  - define net capital holdings as: $a_t^* = \tilde{a}_t - b$
  - substitute government bc into hh bc:
    $\tilde{c}_t + (1+g)\tilde{a}_{t+1}^* \leq (1+r)\tilde{a}_t^* + \omega(r)e_t - \gamma$
  - the new budget constraint is: $\tilde{a}_t^* \geq -b$
  - the new the asset demand is: $\overline{\alpha}^*(r;\gamma,b,g) \equiv \overline{\alpha}(r;\gamma,b,g) - b$
- As debt increases:
  1. **borrowing limit relaxes: enhances consumption smoothing**
     (in addition to saving, hh can borrow to buffer shocks)
  2. **the interest rate rises and capital gets crowded out**
     (smaller capital stock lowers wage and consumption)

REDUCED MODEL - welfare

- What is the welfare effect of an increase in public debt?
  - \+ higher return on assets:
    1. consumption smoothing via savings becomes less costly
    2. and more effective (approach CM equilibrium)
  - — requires increase in lump sum taxation:
    1. higher relative burden for households poor in asset and income
    2. exacerbates relative variability of after-tax earnings
  - — increase in debt crowds out capital (wages, consumption fall)
- AM estimate net utilitarian welfare effect in benchmark model

$$\Omega = \iint V(a, e)\ \mathrm{d}H(a, e)$$

V: optimal value function
H: steady state distribution of assets and productivities
Ω expresses welfare changes in percentage of consumption

BENCHMARK MODEL - in stationary steady state

- Households:

$$\max_{\tilde{c}_t, l_t, \tilde{a}_{t+1}} E\left[(Y_0)^{\eta(1-\mu)} \sum_{t=0}^{\infty} \left[\beta(1+g)^{\eta(1-\mu)}\right]^t \frac{(\tilde{c}_t^{\eta} l_t^{1-\eta})^{1-\mu}}{1-\mu}\right]$$

s.t.

$$\tilde{c}_t + (1+g)\tilde{a}_{t+1} \leq (1+(1-\tau_y)r)\tilde{a}_t + (1-\tau_y)w_t e_t(1-l_t) + \chi$$

$$\tilde{c}_t \geq 0; \tilde{a}_t \geq 0; 1 \geq l_t \geq 0; \tilde{a}_0, e_0, Y_0 \text{ given}$$

- Government: $\gamma + \chi + ((1-\tau_y)r - g)b = \tau_y(1-\delta k)$
- Labor Market: $\overline{\varphi}(r, N; \gamma, b, g, \chi) = N = \mathrm{E}[e_t(1-l_t)]$
- Asset market: $\overline{\alpha}(r, N; \gamma, b, g, \chi) = k + b$

$\Rightarrow$ CE is characterized by r^* and N^*

BENCHMARK MODEL - parametric specification

- Production function: Cobb Douglas (with capital share θ)
- Labor productivity process:
  - assumed to be AR(1)
  - approximated as seven state Markov Chain, Tauchen [1986]
  - from Aiyagari [1994]: $\rho = 0.6$, $\sigma = 0.3$
- Government policies and parameters:
  - averages of US postwar data:
    - $\gamma = 21.7\%$, $\chi = 8.2\%$, $b = 66\%$ (of GDP)
    - $g = 1.85\%$, $\delta = 0.075$, $\theta = 0.3$
  - arbitrary: $\mu = 1.5$
  - match data: $\beta = 0.991$ (align model to empirical interest rate)
  - back out: $\eta = 0.328$ using elasticity of the labor supply of 2%
- $\rho, \sigma, \mu, \beta, \eta$ determine precautionary savings motive
  ($\rightarrow$ govern welfare optimizing amount of debt)

BENCHMARK MODEL - results

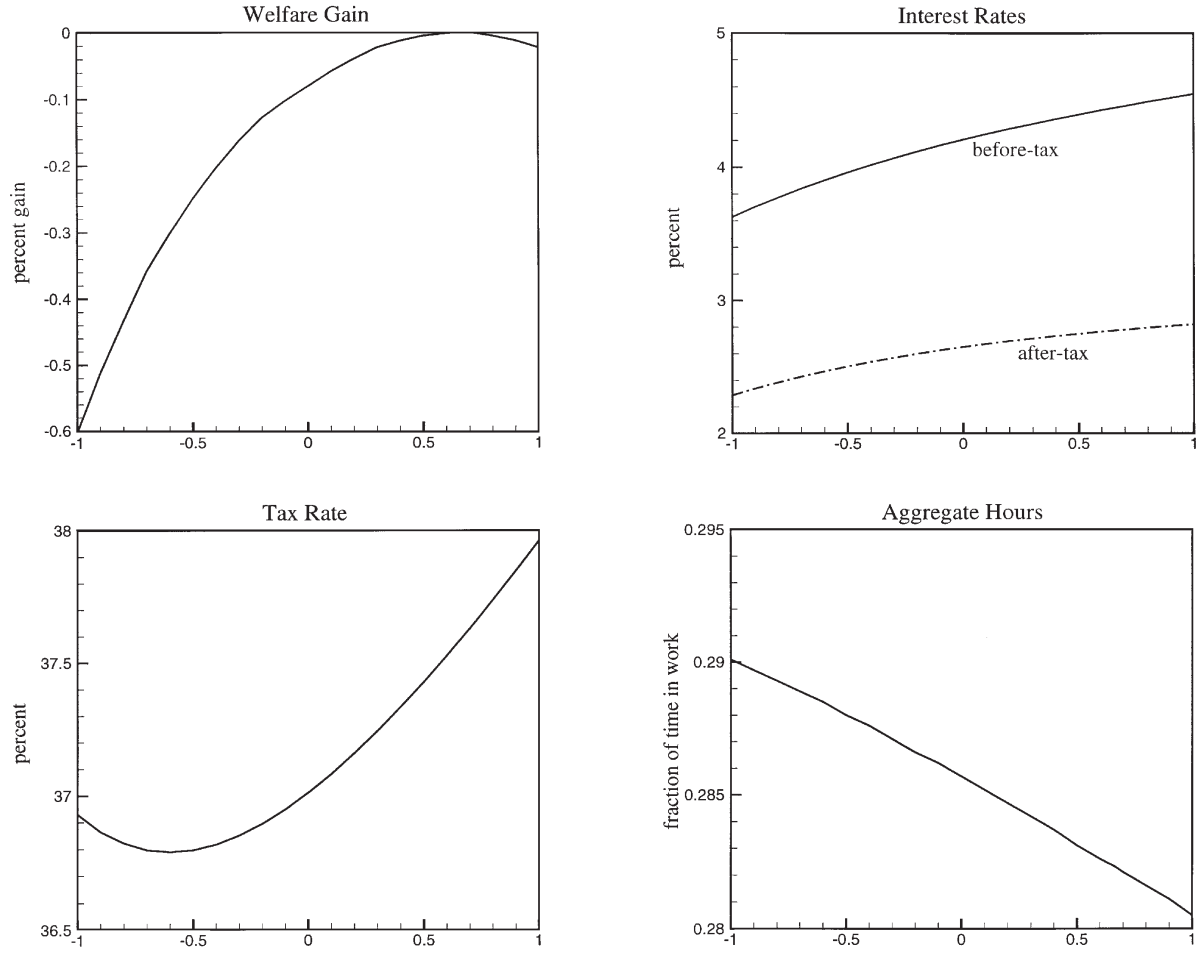

Fig. 2. Welfare gain, interest rates, tax rate, and aggregate hours versus debt/GDP ratio (x-axis) for the benchmark economy.

- Tiny welfare change in debt: positive almost offsets negative effect
- Reduced model: optimal debt is 140%; welfare loss still small

BENCHMARK MODEL - vary parameters to test robustness

Note: AM adjust β simultaneously in robustness tests 1-3

- before-tax interest rate and debt remain at 4.5% and 66%

1. Decrease ρ (σ) $\rightarrow$ optimum amount of public debt is lower
   - reduces asset demand
   - $\star$ Model: optimal debt is 50% (20%); welfare loss negligible
2. Increase $\mu \rightarrow$ effect ambiguous
   - hh more risk averse: wants to smooth more, saves more
   - hh has lower effective discount rate: saves less
   - $\star$ Model: in/decrease lowers optimal debt; welfare loss negligible
3. Increase η to target labor elasticity of 1% $\rightarrow$ effect ambiguous
   - if $\mu > 1$: Larger η lowers effective discount rate: hh saves less
   - increasing η makes labor less elastic so tax less distortionary
   - $\star$ Model: optimal debt is lower; again small welfare loss
4. Adjust β alone to target before-tax interest rate of 6%
   - $\star$ Model: optimal debt -50%; welfare gain 0.48% (of consumption)

CONCLUSION

- AM introduce public debt into an Aiyagari model where it
  - relaxes household borrowing constraints
  - reduces incentives to invest in productive capital and to work
- The model suggests US debt/GDP (66%) is welfare optimal
- This finding is robust to parametric changes in
  - exposure of households to uninsurable labor income risk
  - household preferences (risk aversion, patience, desire to work)